MATHEMATICS DRAFT SAMPLE EXAMINATION
UNITS 3AMAS/3BMAS

Section 7 of the *WACE Manual: General Information 2008 Revised Edition* outlines the policy on WACE examinations.

Further information about the WACE Examinations policy can be accessed from the Curriculum Council website at http://www.curriculum.wa.edu.au/internet/Communications/Publications/.

The purpose for providing a sample examination is to provide teachers with an example of how the course will be examined. Further finetuning will be made to this sample in 2008 by the examination panel following consultation with teachers, measurement specialists and advice from the Assessment, Review and Moderation (ARM) panel.

2008/37889

Disclaimer
Any resources such as texts, websites and so on that may be referred to in this document are provided as examples of resources that teachers can use to support their learning programs. Their inclusion does not imply that they are mandatory or that they are the only resources relevant to the course.

Curriculum Council
Government of Western Australia

WACE
Western Australian Certificate of Education

WACE, Draft Sample Examination 2008

Question/answer booklet

MATHEMATICS 3AMAS/3BMAS

Written paper

Please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this paper

Section One
Reading time before commencing work: 5 minutes
Working time for paper: 50 minutes
Changeover period: approximately 15 minutes
Section Two
Reading time before commencing work: 10 minutes
Working time for paper: 100 minutes

Material required/recommended for this paper

To be provided by the supervisor
Question/answer booklet for Section One, containing a removable formula sheet which may also be used for Section Two

To be provided by the candidate
Section One:
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Section Two:
Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler
Special materials: drawing instruments, templates, notes on up to two unfolded sheets of A4 paper, up to two approved CAS calculators, and one other non-CAS calculator (graphics or scientific)

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available	Percentage
One Calculator—free	8	8	50	40	$33\frac{1}{3}$ %
Two Calculator—allowed	12	12	100	80	$66\frac{2}{3}$ %
			Total marks	120	100%

Instructions to candidates

1. The rules for the conduct of WACE external examinations are detailed in the booklet *WACE Examinations Handbook*. Sitting this examination implies that you agree to abide by these rules.

2. Answer the questions in the spaces provided.

3. Spare answer pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

See next page

Section One: Calculator – free 40 Marks

This section has **eight (8)** questions. Attempt **all** questions.
Suggested working time: 50 minutes

Question 1 **(4 marks)**

Simplify the following:

(a) $\dfrac{a^2 \times \left(3b^2\right)^3}{\left(ab\right)^5}$ (2 marks)

(b) $e^{5\ln x - 3\ln y}$ (2 marks)

Question 2 **(5 marks)**

Find the real and complex parts of the following complex numbers:

(a) $\left(2+3i\right)^2$ (2 marks)

(b) $\dfrac{3+4i}{7-i}$ (2 marks)

(c) i^{99} (1 mark)

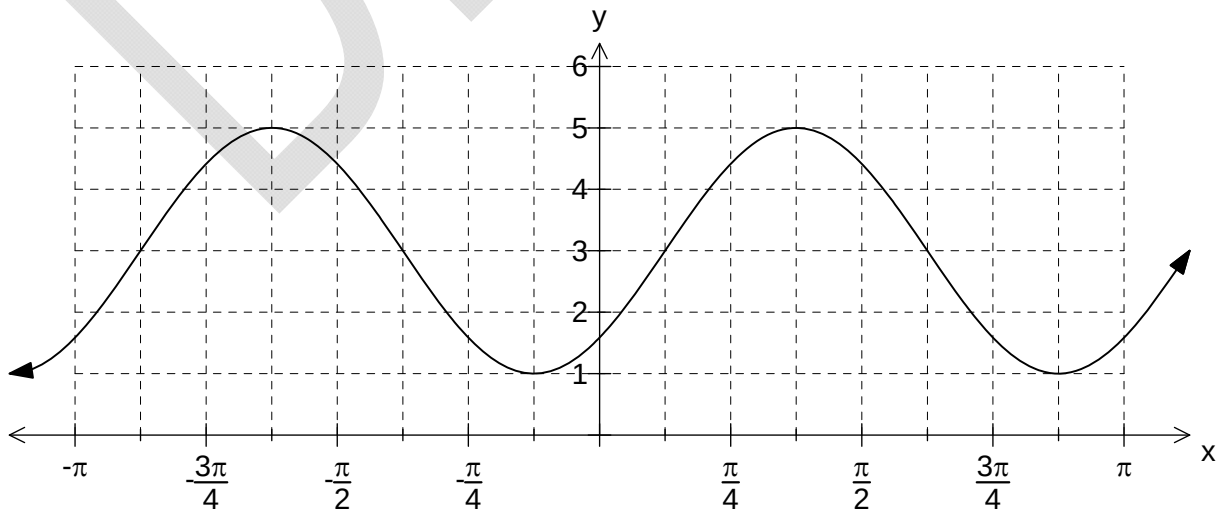
Question 3 **(4 marks)**

Find all values of x in the interval $0 \le x \le \pi$ that satisfy the equation

$$\cos 4x = -\frac{\sqrt{3}}{2}$$

Question 4 **(6 marks)**

Part of the graph of the function $f(x) = a\sin b(x-c) + d$ is shown below. Use the graph to evaluate the constants *a, b, c* and *d, given that* $a > 0, b > 0,$ and $-\pi < c \le \pi$.



See next page

Question 5 **(4 marks)**

Find the derivatives of the following functions. It is not necessary to simplify your answers.

(a) $y = \sqrt{x}(3x-1)^5$ (2 marks)

(b) $f(x) = \ln(x^2 + 3x)$ (2 marks)

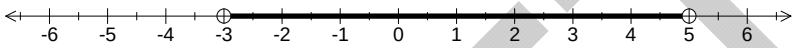
Question 6 **(5 marks)**

Given that tan A = 3 and tan (A + B) = -7, find tan B.

Question 7 **(6 marks)**

(a) The number line below shows the solution set of the inequality $|x-a| < c$.

Determine the values of *a* and *c*. (2 marks)



(b) Solve the inequality $|3x+2| > 4$. (4 marks)

Question 8 **(6 marks)**

The functions $\cosh x$ and $\sinh x$ are defined by

$$\textit{cosh } x = \frac{1}{2}\left(e^x + e^{-x}\right) \quad \text{and} \quad \textit{sinh } x = \frac{1}{2}\left(e^x - e^{-x}\right).$$

(a) Show that $(\textit{cosh } x)^2 - (\textit{sinh } x)^2 = 1.$ (3 marks)

(b) Show that $\frac{d}{dx}(\textit{cosh } x) = \textit{sinh } x.$ (1 mark)

(c) Use your knowledge of the graphs of e^x and e^{-x} to sketch the graph of $\textit{cosh}\, x$. (2 marks)

END OF SECTION ONE

Section Two: Calculator – allowed 80 Marks

This section has **twelve (12)** questions. Attempt **all** questions.
Suggested working time: 100 minutes

Question 9 **(8 marks)**

An orienteer runs 2 km in a north-easterly direction. He then runs 4.5 km due west, followed by 3.6 km at a bearing of 135 degrees i.e. 45 degrees east of south. He now wants to return directly to his starting point.
Use vector methods to determine:

(a) the direction in which he should run;

(b) his distance from the starting point.

Question 10 **(4 marks)**

(a) Find the acute angle between the lines L_1 and L_2 where:

$$L_1: \quad \boldsymbol{r} = \begin{bmatrix} -7 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 15 \\ 8 \end{bmatrix} \qquad L_2: \quad \boldsymbol{r} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} = 19$$

Give your answer in degrees correct to 2 decimal places. (2 marks)

(b) Does the line L_2 pass through the origin? Justify your answer. (2 marks)

Question 11 **(5 marks)**

Perth lies on Latitude 32° South. Given that the radius of the earth is 6350 km:

(a) Find, to the nearest km, the circumference of the circle through which Perth passes during one rotation of the earth on its axis. (2 marks)

(b) Calculate Perth's velocity in km/h. (1 mark)

(c) Comment, without calculation, on how Perth's velocity would compare with that of Jakarta, where the latitude is 6° South. Explain your answer. (2 marks)

Question 12 **(5 marks)**

A and B have polar coordinates (1, α) and (1, β) respectively, where $0^\circ < \alpha < 90^\circ$ and $\beta > \alpha$.

(a) State the coordinates of A and B in Cartesian form. (2 marks)

(b) Write down the scalar product $\overrightarrow{PA} \bullet \overrightarrow{PB}$ (where P is the pole):

(i) using the components of the two vectors;

(ii) using the magnitudes of the vectors and the angle between them. (2 marks)

(c) Hence deduce a formula connecting α and β. (1 mark)

Question 13 **(4 marks)**

Jodie is following a weight-loss program and loses 1% of her weight every week. Jodie's friend Kevin is following an even stricter weight-loss program and loses 2% of his weight every week. If Jodie and Kevin begin their programs at the same time, and Kevin is initially 25% heavier than Jodie, how long will it take until Jodie and Kevin have the same weight?

See next page

Question 14 **(9 marks)**

Julie paddles her kayak in a body of water which is flowing in a south-easterly direction at a speed of 2 km/h. Her speed in still water is 5 km/h.

(a) If Julie paddles due north, in what direction and at what speed does the kayak actually travel? Support your answer with a diagram.

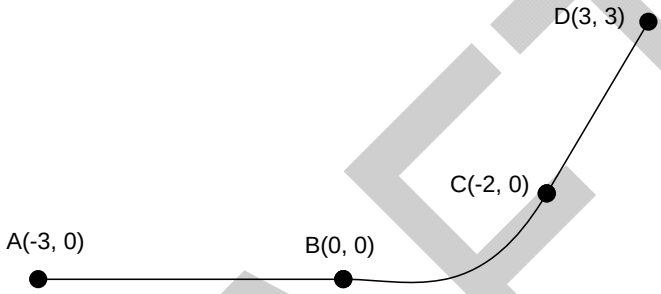
(4 marks)

(b) If Julie wants to travel due north, in what direction should she paddle her kayak, and how fast would she actually travel? Support your answer with a diagram.

(5 marks)

Question 15 **(9 marks)**

The curve ABCD consists of straight line segments AB and CD, and an arc BC whose equation is a cubic polynomial $y = ax^3 + bx^2 + cx + d$



The coordinates of A, B, C and D are as shown in the diagram.

(a) Show that if the curve and its derivative are continuous at B, then $c = 0$ and $d = 0$

(2 marks)

(b) Evaluate a and b, given that the curve and its derivative are continuous at C.

(3 marks)

(c) Where on the curve does the greatest change in the second derivative occur?

(4 marks)

Question 16 **(7 marks)**

The approximation $2^{10} = 1024 \approx 1000 = 10^3$ can be used as a starting point for building a table of estimates of base-10 logarithms.

(a) Take logarithms of both sides of the approximation $2^{10} \approx 10^3$ to show that $\log_{10} 2 \approx 0.3$.

(1 mark)

(b) Use the estimate $\log_{10} 2 \approx 0.3$ to estimate $\log_{10} 4$ and $\log_{10} 8$.

(1 mark)

(c) Now use the approximation $80 \approx 81$ to estimate $\log_{10} 3$ correct to 1 decimal place.

(1 mark)

(d) Use your existing results to estimate $\log_{10} 6$. (1 mark)

(e) The interval [1, 9] can be subdivided into 4 equal sub-intervals. Use your estimates of $\log_{10} 2$, $\log_{10} 4$, $\log_{10} 6$, and $\log_{10} 8$ to estimate $\int_1^9 \log_{10} x \, dx$. (3 marks)

See next page

Question 17 **(12 marks)**

The function f is defined by the formula

$$f(x) = ln\frac{x}{1-x}.$$

(a) What is the domain of the function *f*? (2 marks)

(b) Sketch the graph of f. (2 marks)

(c) Use the graph of f to determine its range. (1 mark)

(d) Use your calculator to find the equation of the tangent to f at $x = \frac{1}{2}$. (2 marks)

(e) Use your sketch of the graph of f to sketch the graph of the inverse f^{-1}. (2 marks)

(f) What are the domain and range of f^{-1}? (2 marks)

(g) If $f(a) = f^{-1}(a)$ what is the value of a? (1 mark)

Question 18 **(4 marks)**

A major sector of a circle has an area of 88 cm^2. If the sector also has a perimeter of 38 cm, determine, exactly, the length of the radius and the angle (in radians) described by the sector.

Question 19 **(6 marks)**

The centre of a circle, C_1, has position vector $\overrightarrow{OA} = -11i + 8j$. The radius of the circle is 15 units.

(a) State the vector equation of C_1. (1 mark)

The centre of a second circle, C_2, has position vector $\overrightarrow{OB} = 9i - 7j$; its radius is 10 units.
If C_1 and C_2 were drawn on a calculator they would appear to touch.

(b) Demonstrate mathematically that C_1 and C_2 do in fact touch. (2 marks)

P is the point of contact between the two circles.

(c) Use the fact that $|\overrightarrow{AP}| : |\overrightarrow{PB}| = 3 : 2$ to determine the coordinates of P. (3 marks)

Question 20 **(7 marks)**

Use trigonometric identities to prove that:

(a) $1 - sin\,2x = (sin\,x - cos\,x)^2$ (3 marks)

(b) $\frac{1-cos\,\theta}{1+cos\,\theta} = tan^2\frac{\theta}{2}$ (4 marks)

End of paper

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